Your Signature ______________________

Instructions:
Please write your name on every page.
Maximum time is 3 hours. Please stop writing when you are asked to do so.

Score

1. (15)	
2. (10)	
3. (15)	
Total. (40)	

1. Let $g : \mathbb{R}^2 \to \mathbb{R}$ be given by

$$g(x) = \begin{cases} \frac{x_1 x_2^3}{x_1^2 + x_2^2} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

Show that g is continuously differentiable at the origin. Calculate $\frac{\partial^2 g}{\partial x_1 \partial x_2}$ and $\frac{\partial^2 g}{\partial x_2 \partial x_1}$ at the origin.

2. Let I be a bounded interval and $f : I \to \mathbb{R}$ be a continuous non-negative Riemann integrable function. Suppose $\int_I f = 0$ then show that $f = 0$ on I.

3. Solve the following questions giving adequate justification.

(a) Suppose $I = [a, b]$ and f is Riemann Integrable on I. Does there exists $c \in (a, b)$ such that $\int_I f = f(c)(b - a)$?

(b) Let α be a positive constant and $g : \mathbb{R}^2 \to \mathbb{R}$ be given by

$$g(x) = \begin{cases} \frac{x_1^\alpha}{x_1^2 + x_2^2} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

Decide whether g is continuous at 0.

(c) Let $x_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $g : \mathbb{R}^2 \to \mathbb{R}^2$ be given by

$$g(x) = \begin{bmatrix} 3x_1^2 \\ 8x_2^2 \end{bmatrix}$$

Decide whether g is differentiable and find $Dg(x_0)$ if it is.